Observing the splitting of the Balmer series on deuterated hydrogen (isotope splitting)

P6.2.1.3 (a) Measurement with ocular and scale

P6.2.1.3 (b) Measurement with VideoCom

KEM 0707

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Observing the splitting of the Balmer series on deuterated hydrogen (isotope splitting)

Objects of the experiments

- Observation of the visible Balmer lines for hydrogen and deuterium.
- Determination of the wavelengths of the visible Balmer lines.
- Observation and estimation of the separation of isotopes.

Principles

The Balmer series of the hydrogen atom is the result of electron transfer to the second main energy level (L shell, principle quantum number n= 2) from higher energy states (m : 3, 4, 5,...). For the frequency or wavelength of the emitted photons the following applies:

$$\nu = \frac{c}{\lambda} = R_\infty \left(\frac{1}{\mathrm{n}^2} - \frac{1}{\mathrm{m}^2}\right) \qquad (1)$$

with the Rydberg constant being

$$R_\infty = \frac{m_e \cdot e^4}{8 \cdot \varepsilon_0^2 \cdot h^3 \cdot c} \qquad (2)$$

For this it is assumed that the mass of the nucleus is very much bigger (∞) than that of the electron ($m_K >> m_e$).

For a precise calculation, the Rydberg constant must be corrected using the reduced mass μ .

$$\mu = \frac{m_e \cdot m_K}{m_e + m_K} = \frac{1}{1 + \frac{m_e}{m_K}} \qquad (3)$$

The result for hydrogen is:

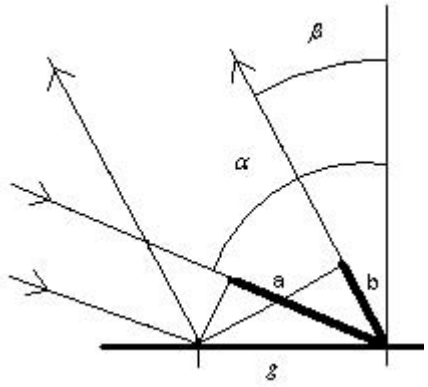
$$R_\mathrm{H} = \frac{R_\infty}{1 + \frac{m_e}{m_p}} \qquad (4)$$

The hydrogen isotope deuterium D consists in its nucleus of a proton and a neutron. The result for the Rydberg constant is:

$$R_\mathrm{D} = \frac{R_\infty}{1 + \frac{m_e}{m_p + m_n}} \qquad (5)$$

The spectral lines in the Balmer series for deuterium are therefore shifted to shorter wavelengths compared to the hydrogen lines. This effect is called isotope splitting and is part of the hyperfine structure.

In the experiment the Balmer lines are investigated by means of a high-resolution spectrometer setup. For this a holographic grating is used. The interference results in the reflection.



The path difference between two beams with 1st order maximum is: $\Delta s = \mathrm{a} + \mathrm{b} = \lambda$

With the incident angle α and the reflected angle β ($\frac{\mathrm{a}}{g} = \sin\alpha$ and $\frac{\mathrm{b}}{g} = \sin\beta$) the wavelength is:

$$\lambda = g \cdot (\sin\alpha + \sin\beta) \qquad (6)$$

with the grating constant g.

The angular separation $\Delta\beta$ of the line is calculated from (6)

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\beta} = g \cdot \cos\beta \text{ or}$$

$$\Delta\lambda = g \cdot \cos\beta \cdot \Delta\beta \qquad (7)$$

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The separation of the lines is observed with a telescope setup. The distance *d* between the lines together with the focal length *f* of the objective allows the determination of the angular separation:

$$\Delta\beta = \frac{d}{f} \qquad (8)$$

This results in a difference between the wavelengths for the isotope splitting:

$$\Delta\lambda = \frac{d \cdot g \cdot \cos\beta}{f} \qquad (9)$$

Apparatus

1 optical bench of standard cross section, 1 m.....460 32
1 optical bench of standard cross section, 0.5 m...460 335
1 rail connection with swivel joint and scale 460 341
6 optics riders 90/50 .....460 374
1 Balmer lamp, deuterated 451 41
1 power supply unit for the Balmer lamp 451 14
1 lens in frame, *f* = 50 mm.....460 02
1 adjustable slit 460 14
1 projection objective.....460 13
1 holographic grating 2400 lines/mm.....471 27
1 lens in frame, *f* = 300 mm.....460 09
1 ocular with scale.....460 135

Setup

Fig. 1: Experimental setup

– First of all align the two optical benches by means of the feet in parallel and at the same height and fix the position using the adjustment screws.
– Screw together the two rails with the swivel joint so that the angular scale is firmly attached to the rail on which the Balmer lamp is mounted.
– Set up the optical elements as shown in the diagram and adjust to the same height.
– Using the *f* = 50 mm lens, form an image of the capillary of the Balmer lamp onto the adjustable slot (distance approx. 10 cm = 2 *f*). If required rotate the Balmer lamp a little.
– Position the projection objective approx. 15 cm behind the adjustable slot.
– Rotate the pillar of the swivel joint in such a way that the pointer points directly to 0° and fix it by means o f the thumb screw.
– Insert the holographic grating vertically so that the light is reflected back to the slot. Slightly shift the projection objective until a crisp image of the slot appears by the side of the slot (auto-collimation) so that the slot is imaged at infinity.
– Adjust the holographic grating by means of the thumb screws so that the image of the slot falls directly on the slot itself.
– Adjust the angle of the optical benches to (e.g.) ω_{benches} =155°.
– Position the ocular (eyepiece) at the end of the rail and adjust in such a way that the scale is easily read.
– Place the *f* = 300 mm lens approx. 30 cm in front of the ocular (telescope setup).

Carrying out the experiment

a) Investigating the red Balmer line

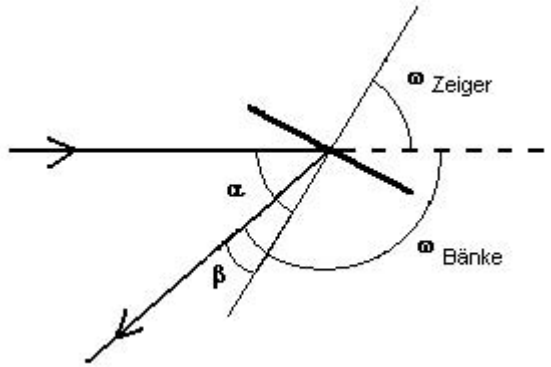
– Loosen the thumb screw on the pillar with the holographic gratings and rotate the pillar until the red Balmer line can be observed. For a better identification of the line, initially open the slot widely and than adjust to approx. 0.1 mm.
– Shift the *f* = 300 mm lens until the image of the slot is crisp.
– Read the angle of the optical benches and the angle of the holographic grating.
– Estimate the distance of separation on the scale of the ocular.

b) Investigation of the other visible Balmer lines

– Rotate the pillar with the holographic grating until the turquoise Balmer line is visible. For better identification of the line initially open the slot widely and than adjust to approx. 0.1 mm.
– Adjust the f = 300 mm lens a little until the image of the slot is crisp again (compensation of the chromatic aberration).
– Read the angle of the optical benches and the angle of the holographic grating.
– Estimate the distance of separation on the scale of the ocular.
– If possible repeat the experiment with the blue and the violet lines. These lines are of lower intensity and are close to the sensitivity of sight, which means that particularly the violet line is very difficult to observe.

Measuring example

a) Investigating the red Balmer line



– The pointer incident angle corresponds to the angle of rotation of the pillar with the holographic grating:

$\alpha = \omega_{\text{pointer}}$

– The angle of reflection β results from:

$$\beta = \omega_{\text{pointer}} + \omega_{\text{benches}} - 180° \qquad (10)$$

Angle between the optical benches	Angle of the holographic grating	Incident angle	Reflection angle
ω_{benches}	ω_{pointer}	α	β
150°	70°	70°	40°

– With (6) and $g = \frac{1}{2.4 \cdot 10^6}\,\text{m} \approx 417\ \text{nm}$ the result is:

$\lambda = 417\ \text{nm} \cdot (\sin 67° + \sin 42°) = 660\ \text{nm}$

Values from literature: $\lambda_{\text{H}\alpha} = 656.28\ \text{nm}$, $\lambda_{\text{D}\alpha} = 656.11\ \text{nm}$

– Separation: *d* = 0.18 mm, with (9) which means

$$\Delta\lambda = \frac{0.2\ \text{mm} \cdot 417\ \text{nm} \cdot \cos 42°}{300\ \text{mm}} = 0.192\ \text{mm}$$

Value from literature: $\Delta\lambda = \lambda_{H\alpha} - \lambda_{D\alpha} = 0.179\ \text{nm}$

b) Investigation of the other visible Balmer lines

– Angle between the rails ω_{benches} =150°

Balmer line		Incident angle	Reflection angle	Wavelength
		α	β	λ
red	α	70°	40°	660 nm
turquoise	β	52°	22°	485 nm
blue	γ	47.5°	17.5°	433 nm
violet	δ	45.5°	15.5°	409 nm

– Values from literature:

– red $\lambda_{\text{H}\alpha} = 656.28\ \text{nm}$, $\lambda_{\text{D}\alpha} = 656.11\ \text{nm}$

turquoise $\lambda_{\text{H}\beta} = 486.13\ \text{nm}$, $\lambda_{\text{D}\beta} = 486.01\ \text{nm}$

blue $\lambda_{\text{H}\gamma} = 434.05\ \text{nm}$, $\lambda_{\text{D}\gamma} = 433.93\ \text{nm}$

violet $\lambda_{\text{H}\delta} = 410.17\ \text{nm}$, $\lambda_{\text{D}\delta} = 410.07\ \text{nm}$

Balmer line		Separation	
		d	$\Delta\lambda$
red	α	0.18 mm	0.19 nm
turquoise	β	0.11 mm	0.14 nm
blue	γ	0.07 mm	0.09 nm

– Values from literature:

$\Delta\lambda_{\text{turquoise}}$ = 0.179 nm

$\Delta\lambda_{\text{turquoise}}$ = 0.132 nm

$\Delta\lambda_{\text{blue}}$ = 0.118 nm

($\Delta\lambda_{violet}$ = 0.112 nm)

Observing the splitting of the Balmer series on deuterated hydrogen (isotope splitting)

Objects of the experiments

- Determination of the wavelengths of the visible Balmer lines.
- Observation and estimation of the separation of isotopes.

Principles

The Balmer series of the hydrogen atom is the result of electron transfer to the second main energy level (L shell, principle quantum number n = 2) from higher energy states (m : 3, 4, 5,...). For the frequency or wavelength of the emitted photons the following applies:

$$\nu = \frac{c}{\lambda} = R_\infty \left(\frac{1}{\mathrm{n}^2} - \frac{1}{\mathrm{m}^2}\right) \qquad (1)$$

with the Rydberg constant being

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For this it is assumed that the mass of the nucleus is very much greater (∞) than that of the electron ($m_K >> m_e$).

For a precise calculation, the Rydberg constant must be corrected using the reduced mass μ :

$$\mu = \frac{m_e \cdot m_K}{m_e + m_K} = \frac{1}{1 + \frac{m_e}{m_K}} \qquad (3)$$

The result for hydrogen is:

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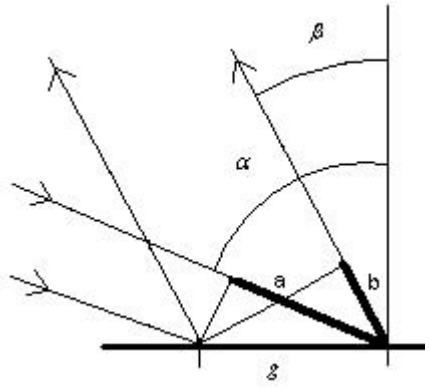
The spectral lines in the Balmer series for deuterium are therefore shifted to shorter wavelengths compared to the hydrogen lines. This effect is called isotope splitting and is part of the hyperfine structure.

In the experiment the Balmer lines are investigated by means of a high-resolution spectrometer setup. For this a holographic grating is used. The interference results in the reflection.



The path difference between two beams with 1st order maximum is: $\Delta s = \mathrm{a} + \mathrm{b} = \lambda$

With the incident angle α and the reflected angle β ($\frac{\mathrm{a}}{g} = \sin\alpha$ and $\frac{\mathrm{b}}{g} = \sin\beta$) the wavelength is:

$$\lambda = g \cdot (\sin\alpha + \sin\beta) \qquad (6)$$

with the grating constant *g*.

The angular separation $\Delta\beta$ of the line is calculated from (6)

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\beta} = g \cdot \cos\beta \text{ or}$$

$$\Delta\lambda = g \cdot \cos\beta \cdot \Delta\beta \qquad (7)$$

The separation of the lines is observed using the VideoCom, whereby the lines are imaged on its row of photo-transistors with a lens of focal length *f* and the difference $\Delta\beta$ in the angles is determined with the “VideoCom Intensities” program.

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Apparatus

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6 optics riders 90/50 .....460 374
1 Balmer lamp, deuterated 451 41
1 power supply unit for the Balmer lamp 451 14
1 lens in frame, *f* = 50 mm.....460 02
1 adjustable slit 460 14
1 projection objective.....460 13
1 holographic grating 2400 lines/mm.....471 27
1 lens in a frame, *f* = 300 mm.....460 09
1 VideoCom USB 337 47USB
add. required: PC with Windows 98 or higher version

Setup

Fig. 1: Experimental setup

– First of all align the two optical benches by means of the feet in parallel and at the same height and fix the position using the adjustment screws.

– Screw together the two rails with the swivel joint so that the angular scale is firmly attached to the rail on which the Balmer lamp is mounted.

– Set up the optical elements as shown in the diagram and adjust to the same height.

– Using the *f* = 50 mm lens, form an image of the capillary of the Balmer lamp onto the adjustable slot (distance approx. 10 cm = 2 *f*). If required rotate the Balmer lamp a little.

– Position the projection objective approx. 15 cm behind the adjustable slot.

– Rotate the pillar of the swivel joint in such a way that the pointer points directly to 0° and fix it by means o f the thumb screw.

– Insert the holographic grating vertically so that the light is reflected back to the slot. Slightly shift the projection objective until a crisp image of the slot appears by the side of the slot (auto-collimation) so that the slot is imaged at infinity.

– Adjust the holographic grating by means of the thumb screws so that the image of the slot falls directly on the slot itself.

– Adjust the angle of the optical benches to (e.g.) ω_{benches} =150°.

– Position the VideoCom at the end of the rail. Place the *f* = 300 mm lens approx. 30 cm in front of the VideoCom.

– Darken the room.

"VideoComInt" program settings (also see operating instructions)

– Connect the VideoCom to the computer and supply with power via the power pack. If necessary install the VideoCom software, and run the "VideoCom Intensities" program.

– Using the  button or the F5 key call the "Calibration/Comparison with theory" menu, and in the "Diffraction Angle" tab enter the effective focal length of 300 mm and close by pressing "OK".

– Using the  button "Longer exposure time" set the exposure time to 8.

– Start the measurement with the  button or the F9 key.

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Carrying out the experiment

a) Investigating the red Balmer line

– Loosen the thumb screw on the pillar with the holographic gratings and rotate the pillar until the red Balmer lines are projected onto the middle of the raw of photo-transistors of the VideoCom. For better visual identification of the line initially open the slot widely and than adjust to approx. 0.1 mm.

– Shift the *f* = 300 mm lens and observe on the monitor until the image of the slit is crisp. If necessary shorting the exposure time or reducing the size of the slit. At 0° two narrow maxima of different heights should be observable.

– Zoom into this area. If necessary set the "Background at Minimum" in the diffraction angle tab.

– For improving the resolution, using the Σ1 button an averaging of the intensities can be started. When the two maxima can be easily distinguished stop the measurement with the button or the F9 key.

– Determine the difference in the angles between the maxima (spectral lines).

– Read the angle of the optical benches and the angle at the holographic grating.

b) Investigation of the other visible Balmer lines

– Rotate the pillar with the holographic grating until the turquoise Balmer lines are projected onto the centre of the photo-transistor line of the VideoCom. For better identification of the line initially open the slot widely and than adjust to approx. 0.1 mm.

– Adjust the f = 300 mm lens a little until the image of the slot is crisp again (compensation for chromatic aberration).

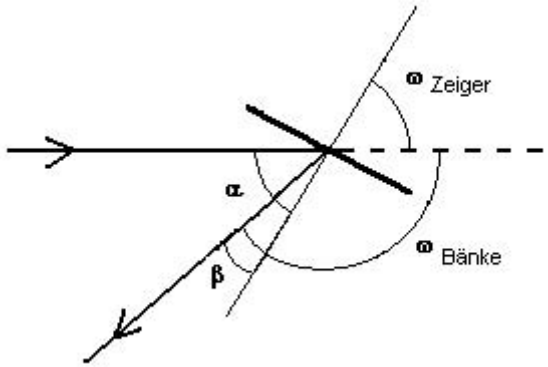
– Determine the angle difference and read the angle of the optical benches and the angle of the holographic grating.

– Repeat the experiment for the blue line. Because of its lower intensity the maxima are less marked.

– The intensity of the violet line is even less, so although it can still be observed, the splitting is barely visible with this setup.

Measuring example

a) Investigating the red Balmer line



– The angle of incidence α corresponds to the angle of rotation of the pillar with the holographic grating: $\alpha = \omega_{\text{pointer}}$.

– The angle of reflection β results from :

$$\beta = \omega_{\text{pointer}} + \omega_{\text{benches}} - 180° \qquad (8)$$

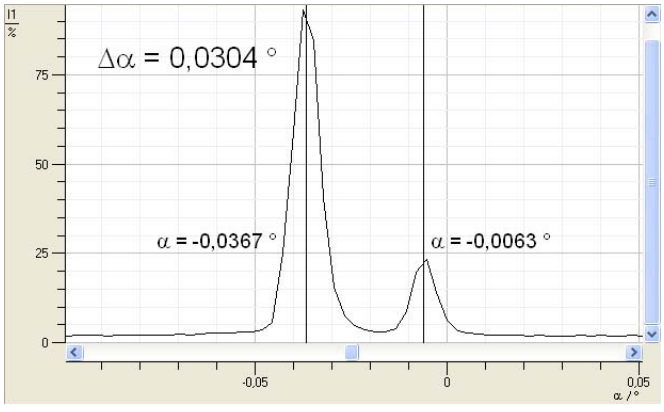
Angle between the optical benches	Angle of the holographic grating	Incident angle	Reflection angle
ω_{benches}	ω_{pointer}	α	β
150°	70.5°	70.5°	40.5°

– With (6) and $g = \frac{1}{2{,}4 \cdot 10^6}\,\text{m} \approx 417\,\text{nm}$ the result is:

$\lambda = 417\,\text{nm} \cdot (\sin 70{,}5° + \sin 40{,}5°) = 664\,\text{nm}$

Values from literature: $\lambda_{\text{H}\alpha} = 656{,}28\,\text{nm}$,

$\lambda_{\text{D}\alpha} = 656{,}11\,\text{nm}$



– Angle difference $\Delta\beta = 0.0304° = 0.000531$

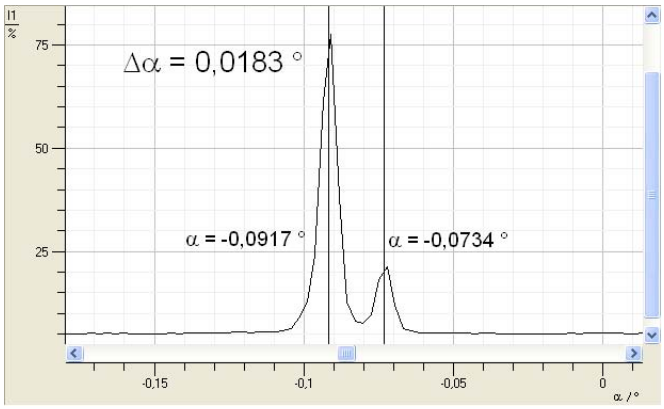
– Using (7), the isotope splitting is

$\Delta\lambda = 417\,\text{nm} \cdot \cos 40{,}5° \cdot 0{,}000531 = 0{,}168\,\text{nm}$

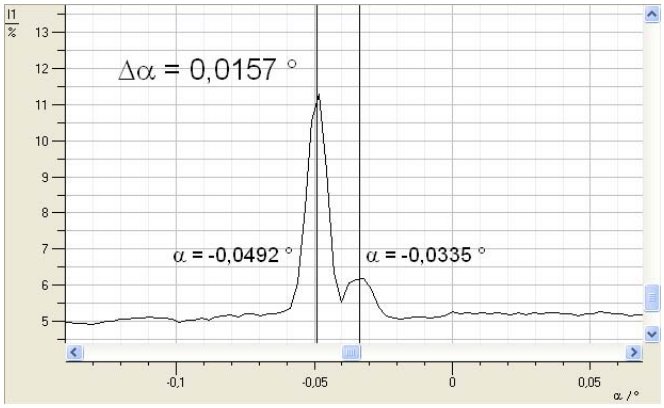
Value from literature: $\Delta\lambda = \lambda_{H\alpha} - \lambda_{D\alpha} = 0{,}179\,\text{nm}$

b) Investigation of the other visible Balmer lines

– *Turquoise line*



– *Blue line*



– Angle between the rails ω_{benches} =150°

Balmer line		Incident angle	Reflection angle	Wavelength
		α	β	λ
red	α	70.5°	40.5°	664 nm
turquoise	β	52.5°	22.5°	490 nm
blue	γ	48°	18°	455 nm
violet	δ	45°	15°	403 nm

– Values from literature:

red $\lambda_{H\alpha} = 656{,}28\,\text{nm}$, $\lambda_{D\alpha} = 656{,}11\,\text{nm}$

turquoise $\lambda_{H\beta} = 486{,}13\,\text{nm}$, $\lambda_{D\beta} = 486{,}01\,\text{nm}$

blue $\lambda_{H\gamma} = 434{,}05\,\text{nm}$, $\lambda_{D\gamma} = 433{,}93\,\text{nm}$

violet $\lambda_{H\delta} = 410{,}17\,\text{nm}$, $\lambda_{D\delta} = 410{,}07\,\text{nm}$

Balmer line		Separation		
		$\Delta\beta$		$\Delta\lambda$
red	α	0.0304°	5.31 10^{-4}	0.168 nm
turquoise	β	0.0183°	3.19 10^{-4}	0.123 nm
blue	γ	0.0157°	2.74 10^{-4}	0.109 nm

– Values from literature:

$\Delta\lambda_{\text{turquoise}}$ = 0.179 nm

$\Delta\lambda_{\text{turquoise}}$ = 0.132 nm

$\Delta\lambda_{\text{blue}}$ = 0.118 nm

($\Delta\lambda_{violet}$ = 0.112 nm)

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